Hausdorff dimension of sets with a common itinerary in exponential dynamics

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In this talk, we discuss the dynamics of exponential maps $f_\lambda \colon \mathbb{C} \to \mathbb{C}$ defined by $f_\lambda(z) = \lambda e^z$, where $\lambda \in \mathbb{C}$. To each point $z \in \mathbb{C}$ we assign an *itinerary*, according to a natural partition of the complex plane related to the periodicity of the map. Using these itineraries, we consider the sets Λ_s consisting of all points that share a given itinerary s. It is known that the escaping points with a given exponentially bounded itinerary form a curve, called a *hair*, that tends to infinity. We are particularly interested in hairs that do not land and whose accumulation set is complicated. In many results it has been shown that, by suitably constructing an itinerary, one can obtain a hair whose closure in the Riemann sphere is an *indecomposable continuum*. This phenomenon arises when a hair becomes so entangled that it accumulates on itself everywhere.

We focus on computing the Hausdorff dimension of the sets Λ_s under the assumption that $\lambda = 1$. Since, for exponentially bounded itinerary s, Λ_s contains a hair, that is, a smooth curve, its Hausdorff dimension is at least 1. We show that for every exponentially bounded itinerary it equals exactly 1. This is joint work with Joanna Horbaczewska and Łukasz Pawelec.