Meta-stability in SDE related to simple climate models: model selection; the light tail limit of Lévy noise

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Abstract

Interpreting paleo-climatic time series by simple dynamical systems with noise leads to statistical model selection problems. For instance, one needs an efficient testing method for the best fitting $\alpha$-stable noise component. We develop a statistical testing method based on the $p$-variation of the solution trajectories of SDE with Lévy noise, for example by showing asymptotic normality or asymptotic $\beta$-stability of their approximations along finite interval partitions.

It has been suggested that the exit and transition characteristics of dynamical systems perturbed by small Lévy noise approach Gaussian behavior as the heavy tails of their jump laws become exponentially light of order $\gamma$, i.e. if for $x \to \infty$ they are given by $\exp(-cx^\gamma)$, and as $\gamma \to 2$. We show that this is surprisingly false, by exhibiting an intriguing phase transition at $\gamma = 1$. 