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Mathematical model of Wood Frog Population

The aim of this work is to embed into a mathematical model the Wood Frog, *Rana sylvatica*, population data collected by Berven, [3], over more than 25 years. The life cycle of the frogs includes aquatic and terrestrial phases, and the competition in each phase is for different resources. Hence, we deal with separate populations, each one providing the new recruits for the other one, see, e.g., [1]. In the case of the Wood Frogs, there are three main stages of development where the individuals compete for different resources. The toads live in the water, and following their metamorphosis they become juvenile frogs, not yet large enough to reproduce. The third stage is of mature egg laying frogs. The populations in these three stages of development have different dynamics. Hence, they are modelled with different mathematical tools, which makes assembling the model an interesting mathematical problem. Due to the seasons in Michigan, the eggs are laid over a short time period and the juveniles emerge from the water more or less the same time, so, we model these two events by impulses, [4]. The success of the metamorphosis depends mainly on the size of the toads. Hence, the size distribution of toads at the time of metamorphosis determines both the number of juveniles and their initial size. Similarly, the transfer from juvenile to adults depends mainly on the size of the frogs. It does not occur at a fixed time, and the juveniles who do not grow sufficiently to mate need to wait for a year before laying eggs. The growth of the toads and the juveniles in size is not uniform across the population and depends on external factors, as well. It is modelled using PDEs for the density size distribution at time t. The death and fertility rates of mature frogs are not related to their age. So their population is assumed to be homogeneous and is modelled by an ODE. Thus, the derived model comprises a system of ordinary and partial impulsive differential equations. The mathematical analysis of such a model can be complicated, see, e.g., [1]. Our analysis and numerical simulations focus on the global properties of the model as a dynamical system, as in [2]. The results show that the model may have a unique solution that converges to a stable periodic cycle.

References

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