From a PDE model to an ODE model of dynamics of synaptic depression

We provide a link between two recent models of dynamics of synaptic depression. To this end, we correct the erroneous boundary condition and specify the missing transmission conditions in the PDE model of Bielecki and Kalita, and show that as the diffusion coefficients tend to infinity and the relative permeability coefficients of the membranes involved tend to zero, the solutions to the PDE model tend to those of the original ODE model of Aristizabal and Glavinovič. Hence, from the mathematical point of view the ODE model is obtained as a singular perturbation of the PDE model with singularities both in the operator and in the boundary and transmission conditions. The result is therefore conveniently put in the context of degenerate convergence of semigroups of operators, where a sequence of strongly continuous semigroups approaches a semigroup that is strongly continuous only on a subspace of the original Banach space. Biologically, our approach allows a new, natural interpretation of the ODE model’s parameters.