Tilmann Glimm
Western Washington University
e-mail: glimmt@wwu.edu

Pattern formation in reaction-diffusion systems with an external morphogen gradient

Gradients of signalling molecules are abundant in the early embryo. They are central to early development. The Turing mechanism in reaction-diffusion systems is a paradigm for pattern formation which has been proposed as an explanation for many developmental phenomena. We propose a generic model of a reaction-diffusion system consisting of an activator and an inhibitor molecule in the presence of a linear morphogen gradient. We assume that this morphogen gradient is established independently of the reaction-diffusion system. Hence it is referred to as an "external" morphogen. It acts by increasing the production of the activator proportional to the morphogen concentration. The model is motivated by several existing models in developmental biology in which a Turing patterning mechanism is proposed and various chemical gradients are known to be important for development. Mathematically, this leads to reaction-diffusion equations with explicit spatial dependence. We investigate how the Turing pattern is affected, if it exists. We also show that in the parameter range where a Turing pattern is not possible, the system may nevertheless produce "Turing-like" patterns. We also apply our general findings to a model of bone pattern formation in vertebrate limbs and show how they may shed light on some experimental findings concerning the action of the protein *Sonic Hedgehog*.

References

[1] T. Glimm, J. Zhang and Y.-Q. Shen *Interaction of Turing patterns with an external linear morphogen gradient* Nonlinearity **22** 10, 2541-2560 (2009).

[2] T. Glimm, J. Zhang, Y.-Q. Shen and S. A. Newman *Reaction-diffusion systems and external morphogen gradients: The two-dimensional case, with an application to skeletal pattern formation* submitted (2010).

[3] M. Alber, T. Glimm, H.G.E. Hentschel, B. Kazmierczak, Y.-T. Zhang, J. Zhu and S. A. Newman *The morphostatic limit for a model of skeletal pattern formation in the vertebrate limb* Bull. Math. Biol. **70** 460–483 (2008).