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## Lumped models for tumor progression

(Primary)tumors have been described mainly as localized entities which grow by mitotic duplication (with a given intrinsic maximal growth rate) in restricted conditions. Such restrictions will slow tumor growth rate until a proper value of carrying capacity is reached.

Some of the most popular scenarios, reflecting tumor growth in specific phases of development ( avascular phase, 'multipassage'syngenic transplant in mice, development of the necrotic core, angiogenesis, invasive phase,..)can be satisfactorily described by means of the Phenomenological Universality (PUN) method, which assumes that the tumor volume  $V$  depends on the growth rate  $c(t)$ , whose effective time derivative can be approximated by a series expansion in the variable  $c(t)$  itself:

$$dV/dt = c(t) V; dc/dt = -\alpha c - \beta c^2 + \dots$$

Retaining only the constant term we get the unlimited growth  $U(0)$ , while by considering the linear term the Gompertz law  $U(1)$  is obtained, accounting for a time-varying growth rate and a constant carrying capacity. $U(2)$ , which is the following term, corresponds to the so called West law, whose main characteristics is that of accounting for tumor vascularization through an 'optimal' fractal network. As a matter of fact,  $U(2)$  entails a variation in the overall tumor carrying capacity, that in a more general sense becomes not only dependent from the limiting volume for tumor development, but on the overall environmental conditions, including nutrients availability, switch to different metabolic pathways, hormonal influences and so on.

Provided the two main parameters, i.e. growth rate and carrying capacity, are modulated in time to properly account for the internal metabolism and the relationship between the tumor and its environment respectively, a full description of the 'natural history' of the tumor can finally be obtained. Comparison with available data and clinical description ( e.g. for the case of prostate cancer) will help in finely modulating the model parameters. Even more interestingly, such a general model is suitable for 'theoretical' validation of therapeutic efficiency. The effect of therapy  $t(t)$ , whose functional form can be expressed in terms of tumor radiosensitivity, drug resistance, etc., can be incorporated into Eqn. 1 by substituting  $c(t)$  with the difference  $c(t) - t(t)$ . Spatially inhomogeneous tumor patterns can be included provided different 'clones' of cells are accounted for.

In conclusion, by retaining the tumor biological complexity in the progressively changing values of the growth rate and carrying capacity of the tumor-host system, a easy-to-handle lumped-model can be worked out, which can prove useful to further stimulate and improve cooperations between theoreticians and clinicians.