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Measure-transmission conditions - a powerful tool in modeling bimodal dynamics

Differentiation of cells may be subject to two paradigms. Either a cell is in a state of inevitable alteration of its characteristics or the state is quasi-stationary, meaning that for a certain period of time the biochemical characteristics remain the same. A cell in the former, transient state usually originated in and heads towards the latter, reaching it in a finite time. On the other hand, a cell in a quasi-stationary state may stay there arbitrarily long and is typically capable of both self-renewal (by division) and differentiation (with or without division). Incidentally, all these scenarios may coincide in a single system, as e.g. in the case of neurogenesis, and lead to interesting bimodal dynamics. These two types of dynamics can be modeled by transport equations or (a system of) ordinary differential equations, respectively. Nonetheless, the two approaches can be unified in a purely continuous setting of measure-valued solutions of the transport equation with additional transmission conditions. In the simplest case, this leads to the following problem ([1]):

$$\begin{aligned}
&\partial_t \mu(t) + \partial_x (g(v(t)\mathbf{1}_{x\neq x_i}(x)\mu(t)) = p(v(t),x)\mu(t),\\
&\qquad\qquad g(v(t))\frac{d\mu(t)}{d\mathcal{L}^1}(x_i^+) = c_i(v(t))\int_{\{x_i\}} d\mu(t), \quad i = 0,\ldots,N,\\
&\qquad\qquad \mu(0) = \mu_0,\\
&\qquad\qquad v(t) = \int_{\{x_N\}} d\mu(t).
\end{aligned}$$

In the talk, we present this new setting and discuss how it allows to capture in an elegant way a wealth of effects, promising interesting applications well beyond its original motivation.

References

[1] Piotr Gwiazda, Grzegorz Jamróz, Anna Marciniak-Czochra, *Models of discrete and continuous cell differentiation in the framework of transport equation*. Submitted.