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Existence and Asymptotic Behaviour of Solutions to Nonlinear Evolution Equations Arising in Mathematical Models of Tumour growth

In this talk we investigate the global existence in time and asymptotic profile of the solution of nonlinear evolution equations with strong dissipation. Applying the above result to some models of mathematical biology and medicine, we discuss mathematical properties of them.

For this purpose we first show the solvability and the asymptotic profile of the solution to the intial boundary value problem of non linear evolution equations:

$$(\mathrm{NE})\begin{cases} u_{tt} = D\nabla^2 u_t + \nabla\cdot(\chi(u_t, e^{-u})e^{-u}\nabla u) & \text{in } \mathbf{\Omega}\times(0,T) \quad (1.1)\\ \dfrac{\partial}{\partial\nu}u\,|_{\partial\mathbf{\Omega}} = 0 & \text{on } \partial\mathbf{\Omega}\times(0,T) \quad (1.2)\\ u(x,0)=u_0(x),\quad u_t(x,0)=u_1(x) & \text{in } \mathbf{\Omega} \quad (1.3)\end{cases}$$

where Ω is a bounded domain in R^n and $\partial\Omega$ is a smooth boundary of Ω and ν is the outer unit normal vector and we denote

$$\frac{\partial}{\partial t}=\partial_t,\ \frac{\partial}{\partial x_i}=\partial_{x_i},\ i=1\cdots,n,\ \nabla u=\mathrm{grad}_x u=(\partial_{x_1}u,\cdots,\partial_{x_n}u)$$

$$\nabla^2 u=\nabla\cdot\nabla u=\Delta u=\partial_{x_1}^2 u+\cdots+\partial_{x_n}^2 u.$$

(1.1) includes the nonlinear evolution equations considered in [4]-[6] to show the global existence in time and the asymptotic profile of the solution of the corresponding mathematical models. We improve our mathematical approach and obtain the solution of (NE), which is in general form of one obtained in them. Next we apply our result to mathematical models of tumour growth, tumour induced angiogenesis and tumour invasion, proposed by Chaplain and Anderson(see [1]-[3]).

References

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