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Delayed protein degradation does not cause oscillations

It is well known that time delays may cause oscillations in solutions of ordinary differential equations. We would like to point out that the presence of oscillations depends on particular causes of a time delay.

Models with time delays may be divided into two families [1,2]. In social-type models, where individuals react to the information concerning the state of the population at some earlier time, we should expect oscillations. On the other hand, in biological-type models, where some changes already take place in the population at an earlier time, oscillations might not be present for any time delay. We will briefly review two specific examples of evolutionary games - replicator dynamics with time delay [1].

Our main goal is to show that delayed degradation does not cause oscillations as it was recently argued [3]. To do so we propose a new methodology to deal with time delays in biological systems and apply it to simple models of gene expression with delayed degradation [4].

We develop a systematic analytical treatment of stochastic models of time delays. Specifically, we take into account that some reactions, for example degradation, are consuming, that is once molecules start to degrade they cannot be part in other degradation processes. It follows from our rigorous analysis that one should look for different mechanisms than just delayed protein degradation to explain causes of oscillations observed in certain biological experiments.

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