Filip Strobin

On a certain generalization of the iterated function systems

Let (X,d) be a metric space. If $f_1,...,f_n : X \to X$ are continuous, then the system $\mathcal{S} = (f_1,...,f_n)$ is called an *iterated function system* (IFS in short). The classical Hutchinson-Barnsley theorem from early 80's states that if X is complete and $f_1,...,f_n$ are Banach contractions (i.e., the Lipschitz constants $Lip(f_i) < 1$), then there is a unique nonempty and compact set $A_{\mathcal{S}} \subset X$ (called a *fractal* or *attractor generated by* $\mathcal{S}$) such that

$$A_{\mathcal{S}} = f_1(A_{\mathcal{S}}) \cup ... \cup f_n(A_{\mathcal{S}})$$

In 2008 Miculescu and Mihail introduced a generalization of the notion of iterated function systems. Namely, instead of selfmaps of a metric space X, they considered mappings $f_i : X^m \to X$, where X^m is the Cartesian product of m copies of X, and $m \in \mathbb{N}$ is fixed. It turned out that the systems of such mappings (called GIFSs) can generate unique fractal sets $A_{\mathcal{S}}$ in the sense of the condition

$$A_{\mathcal{S}} = f_1(A_{\mathcal{S}} \times ... \times A_{\mathcal{S}}) \cup ... \cup f_n(A_{\mathcal{S}} \times ... \times A_{\mathcal{S}})$$

and the fractal theory can be developed also in this setting.

During the talk I will present basic results on GIFSs and GIFS's fractals. In particular, I will show the counterpart of the H-B theorem, present the code space for GIFSs and construct a Cantor set which is a fractal generated by some GIFS, but cannot be obtained as an attractor of any IFS.