Description
We consider a one side linear cocycle. Oseledets theorem shows that Lyapunov exponents exsit for a.e. point. In addition, it demonstrates that Lyapunov exponents are somehow related to an invariant measure. We are going to go deep in that direction. We will state Furstenberg theorem and prove it.
The theorem says that one can compute maximal Lyapunov exponents from an invariant measure of convenient system. If one assumes strong irreducibility then the invariant measure is equal to $\mu \times \eta$ such that μ is an invariant measure of the base map and η on $\mathbb{PR}^d$ is stationary measure of the cocycle. Moreover, η is unique and nonatomic. Such η is called the Furstenberg measure.

References

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