Emergence of non-ergodic dynamics.

Recently we showed the existence of a locally generic family of maps which display the Newhouse phenomena at every parameter. To quantify the complexity of the ergodic behavior of such maps f, I defined the emergence $\mathcal{E}(\epsilon)$ as the minimal number N of probability measures $(\mu_i)_{1\le i\le N}$ necessarily so that the empirical function $x \mapsto E_k(x) := \frac{1}{k}\sum_{i=1}^{k} \delta_{f^i(x)}$ satisfies:

$$\limsup_{k\to\infty} \int_M d_{W_1}(E_k(x), \{\mu_i : 1 \le i \le N\}) dLeb < \epsilon ,$$

where d_{W_1} is the 1-Wasserstein metric on the space of probability measures.

Conjecture 1 (B. 2017)**.** *Many kinds of typical dynamics displays a super polynomial emergence: for every $k > 0$, $\limsup_{\epsilon\to 0} \mathcal{E}(\epsilon)\cdot \epsilon^k \to \infty$.*

I will present some recent progress toward this conjecture in differentiable, Hamiltonian or holomorphic dynamics.