**Title:** *Semigroups of hyperbolic isometries and their parameter spaces*

**Abstract:** Motivated by the theories of Kleinian groups and of $SL(2,\mathbb{C})$-valued linear cocycles, we study semigroups of hyperbolic isometries and their parameter spaces.

In the first part of the talk we consider such semigroups for which the identity is not an accumulation point, and describe these semigroups as *semidiscrete*. The semidiscreteness property on semigroups appears to play a role similar to the discreteness property on groups, and we find that several theorems on Fuchsian groups have semidiscrete counterparts. This is joint work with Ian Short.

In the second part we consider the parameter space of semigroups of hyperbolic isometries. For any point $(f_1, \ldots, f_d)$ in the parameter space $SL(2,\mathbb{C})^d$, each sequence $(i_1, \ldots, i_n, \ldots)$ over the finite set $\{1,\ldots,d\}$ gives rise to an orbit of points in 3-dimensional hyperbolic space, with $n^{\text{th}}$ term given by $f_{i_1} \ldots f_{i_n}(0)$. For a.e. such sequence this orbit escapes to the ideal boundary at a uniform rate, defined as the Lyapunov exponent, λ, of $(f_1, \ldots, f_d)$. We consider the minimum rate of escape, η, and define the *hyperbolic locus*, H, as the set of points in parameter space where η is positive. In 2009, Avila showed that λ is harmonic on H and subharmonic everywhere. We show that η is superharmonic on H, and deduce that H is dense in $SL(2,\mathbb{C})^d$. This is in sharp contrast to the situation in the smaller parameter space, $SL(2,\mathbb{C})^d$, as studied in a 2010 paper of Avila, Bochi and Yoccoz. We finish by answering an open problem from that paper.