GENERALIZED CALABI TYPE KÄHLER SURFACES.

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Abstract. *QCH Kähler surfaces* are Kähler surfaces $(M, g, \bar{J})$ admitting a global, 2-dimensional, $\bar{J}$-invariant distribution $\mathcal{D}$ having the following property: The holomorphic curvature $K(\pi) = R(X, \bar{J}X, \bar{J}X, X)$ of any $\bar{J}$-invariant 2-plane $\pi \subset T_xM$, where $X \in \pi$ and $g(X, X) = 1$, depends only on the point x and the number $|X_{\mathcal{D}}| = \sqrt{g(X_{\mathcal{D}}, X_{\mathcal{D}})}$, where $X_{\mathcal{D}}$ is the orthogonal projection of X on $\mathcal{D}$. In the our talk we obtain a classification of generalized Calabi type Kähler surfaces. By a *generalized Calabi type Kähler surface* we mean a QCH Kähler surface such that the opposite almost Hermitian structure J is determined by a complex foliation into complex curves and is Hermitian. It just means that $\mathcal{D}$ is integrable and J is also integrable. In particular we obtain, by a different method than in Apostolov, Calderbank and Gaudochon a classification of Calabi type Kähler surfaces. We introduce a special orthonormal frame $\{E_1, E_2, E_3, E_4\}$ and dual coframe $\{\theta_1, \theta_2, \theta_3, \theta_4\}$. We also find local coordinates (x, y, z, t) in which $\theta_1 = f dx$, $\theta_2 = f dy$ and $E_4 = \partial_z, E_3 = \beta(z)\partial_t$ where β is some smooth function and $\alpha = \pm\frac{1}{2\sqrt{2}}|\nabla J|$ depends only on z. We show that such coordinates always exist. We classify generalized Calabi type Kähler QCH surfaces which admit an opposite Hermitian structure which is not locally conformally Kähler. In our talk we exhibit a large class of Hermitian surfaces (M, J) with degenerate Weyl tensor W^+ and J-invariant Ricci tensor, which are not locally conformally Kähler. The generalized Calabi type Kähler surfaces which are not of Calabi type are fiber bundles over a Riemannian surface Σ and the fibers which are leafs of the foliation $\mathcal{D}$ have constant sectional curvature : 0 in the semi-symmetric case, positive $4a^2$ if $\alpha = 2a \tan az$ and negative $-4a^2$ if $\alpha = -2a \coth az$ or $\alpha = -2a \tanh az$.