CHROMATIC ZEROS ON HIERARCHICAL LATTICES AND EQUIDISTRIBUTION ON PARAMETER SPACE

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Abstract. Associated to any finite simple graph $\Gamma = (V, E)$ is the *chromatic polynomial* $P_\Gamma(q)$, which has the property that for any integer $k \geq 0$, $P_\Gamma(k)$ is the number of ways to properly colour the vertices of Γ using k colours. The degree of $P_\Gamma(q)$ is $|V|$. A hierarchical lattice is a sequence of graphs $\{\Gamma_n\}_{n=0}^{\infty}$ built recursively under a generating graph. For each $n \geq 0$, let μ_n be the probability measure

$$\mu_n := \frac{1}{|V_n|} \sum_{\substack{q \in \mathbb{C} \\ P_{\Gamma_n}(q)=0}} \delta_q.$$

We prove that if the generating graph is 2-connected, then the sequence of measures μ_n converges to some measure μ, called the *limiting measure of chromatic zeros for* $\{\Gamma_n\}_{n=0}^{\infty}$. For the Diamond Hierarchical Lattice (DHL), we show that its limiting measure has Hausdorff dimension 2.

The main techniques come from holomorphic dynamics, in particular we prove a new equidistribution result that relates the chromatic zeros of a hierarchical lattice to the bifurcation/activity current associated to a particular marked point. This is joint work with Roland Roeder.

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