From invariant ergodic measures to indecomposable characters on full groups.

One of many relations between Dynamical Systems, Ergodic Theory and Representation Theory is a correspondence between measure-preserving group actions and characters (positive-definite conjugation invariant functions) on a group in which the action (μ, X, G) is mapped to the character χ on the group G given by

$$\chi(g) = \mu(\{x \in X : gx = x\}). \tag{0.1}$$

Typically, this correspondence is far from being surjective. However, Anatoly Vershik suggested that for a "sufficiently rich" simple group G every indecomposable character (extreme point in the space of characters) on G can be obtained by formula (0.1) from some ergodic measure-preserving action of G. In my talk I will explain this correspondence for certain full groups and outline the proof of Vershik's conjecture for them under some natural restrictions. The talk is based on an ongoing joint work with Konstantin Medynets.