Iteration and iterative roots of continuous self-maps

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Abstract

The dynamical system of a continuous self-map is generated by iteration of the map; however, the iteration itself, being an operator on the space of continuous self-maps, may generate interesting dynamical behaviors. In this talk, we use the Babbage functional equation $g^n = \mathrm{id}$ to investigate some dynamical properties of the iteration operators on the space $\mathcal{C}(X)$ of continuous self-maps of a locally compact Hausdorff space X. On the other hand, we also discuss a new result on the nonexistence of solutions for the iterative root problem $g^n = f$, a general form of the Babbage functional equation, on arbitrary sets and use it to prove that every nonempty open set of $\mathcal{C}(X)$ contains a map that does not have even discontinuous iterative roots of order $n \geq 2$ for $X = [0,1]^m$, $\mathbb{R}^m$ and S^1. This, in particular, proves that continuous self-maps on X with no continuous iterative roots at all are dense in $\mathcal{C}(X)$ for these X.