TITLE: ASYMPTOTIC PAIRS IN TOPOLOGICAL ACTIONS OF COUNTABLE AMENABLE GROUPS

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Abstract:

By the known theorem of F. Blanchard, B. Host and S. Ruette, every topological $\mathbb{Z}$-action of positive entropy admits asymptotic pairs. Moreover, T. Downarowicz and Y. Lacroix proved that every topological $\mathbb{Z}$-action of entropy zero has an extension with no asymptotic pairs. Together, these two results give a characterization of zero-entropy topological $\mathbb{Z}$-actions as factors of systems with no asymptotic pairs. Recently developed theory of multiorders allowed us to achieve a similar characterization for topological actions of countable amenable groups.

We begin the talk by introducing the notion of a multiorder and present some basic properties of multiorders on countable amenable groups. Then, we provide a definition of a $\prec$-asymptotic pair in a topological action (X, G) of a countable amenable group G. In the case where for some G-invariant Borel probability measure μ on X, the measure-preserving system (X, μ, G) factors, via a map φ, onto some multiorder $(\tilde{\mathcal{O}}, \nu, G)$, we also introduce the notion of a φ-asymptotic pair. Then we prove that if μ has positive measure-theoretic conditional entropy with respect to the multiorder factor, then the set of points which belong to φ-asymptotic pairs has positive measure μ. As a strengthening of this result, we show that for any system (X, G) of positive topological entropy, any multiorder $(\tilde{\mathcal{O}}, \nu, G)$ and ν-almost every $\prec \in \tilde{\mathcal{O}}$, there exist $\prec$-asymptotic pairs in X. Both of these results generalizes the classical Blanchrd-Host-Ruette Theorem. Finally, we characterize systems (X, G) of topological entropy zero as factors of topologically multiordered systems with no φ-asymptotic pairs.

The talk is based on the joint work with Tomasz Downarowicz.