Lorenz Maps: Renormalizations, Periodic Orbits, and the Connection to the Lorenz System

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Lorenz maps are interval maps with a single discontinuity, which appear in a natural way as Poincaré maps in geometric models of the well-known Lorenz attractor. In the first part of the talk, we will focus on renormalizations of Lorenz maps, i.e., certain return maps of a Lorenz map to smaller intervals around the discontinuity. The existence of renormalizations is closely related to the existence of completely invariant sets. This relation, however, turns out to be more delicate than the results in the literature indicate. It was believed that each renormalization corresponds to some completely invariant set that defines it, but in fact, for some of them, such a set does not exist. Motivated by this observation, we provide an algorithm to distinguish the renormalizations that can and cannot be recovered from completely invariant sets, and we reveal the essence of the difference between them. These results are based on joint work with Piotr Oprocha.

In the second part of the talk, we will shift our attention to the applications of Lorenz maps in the study of the Lorenz system. We will discuss the relation between the first return map of the attractor (for certain parameters of the Lorenz system) and a suitable symmetric Lorenz map with a constant slope (the so-called β-transformation). Using both analytical and symbolic approaches, we will describe the periodic structure of symmetric β-transformations, and then we will show how the obtained information can be transferred to the attractor. These results were obtained together with Eran Igra.