

Title: Coexistence of mixing and rigid behaviors of probability preserving transformations.

Abstract: Motivated by a conjectured refinement of the polynomial Szemerédi theorem, we introduce the concept of *group of rigidity* (associated with a finite family of sequences). This new concept allows us to turn questions about the asymptotic coexistence of mixing and rigid behaviors of a probability preserving transformation into simpler algebraic conditions. This algebraic reduction enables us to construct probability preserving systems relevant to this conjecture. Furthermore, groups of rigidity play an instrumental role in answering questions stemming from the theory of generic Lebesgue preserving automorphisms of $[0, 1]$, IP-ergodic theory, multiple recurrence, and spectral theory. As an illustration, we present the following characterization of when arbitrary combinations of rigidity and mixing behaviors can be realized along integer multiples of a common sequence:

Given $(b_1, \dots, b_\ell) \in \mathbb{N}^\ell$, there is no integer vector orthogonal to $(b_1, \dots, b_\ell)$ having a coordinate of absolute value 1 if and only if there is an increasing sequence $(n_k)_{k \in \mathbb{N}}$ in $\mathbb{N}$ with the property that for every $F \subseteq \{1, \dots, \ell\}$ there is a μ -preserving transformation $T_F : [0, 1] \rightarrow [0, 1]$ (μ denotes the Lebesgue measure) such that

$$\lim_{k \rightarrow \infty} \mu(A \cap T_F^{-b_j n_k} B) = \begin{cases} \mu(A \cap B), & \text{if } j \in F, \\ \mu(A)\mu(B), & \text{if } j \notin F, \end{cases}$$

for every pair of measurable sets $A, B \subseteq [0, 1]$. Part of this talk is based on joint work with Vitaly Bergelson.