The classical Brjuno function is a fundamental object in the study of small divisors and the stability of holomorphic maps. An interesting variant, introduced by Marmi-Moussa-Yoccoz, replaces the standard logarithmic singularity with a power-law divergence $x^{-1/\sigma}$ ($\sigma > 0$). While lower semi-continuity guarantees the existence of a global minimum, the fractal nature of these functions makes identifying the precise location of these minimizers a significant challenge.

In recent joint work with Carlo Carminati and Stefano Marmi, we provide a rigorous characterization of these minima. We prove that for any integer $\sigma = n$, the unique global minimum is attained at the point with continued fraction expansion $[0; \overline{n+1}]$. Furthermore, we establish a local stability result: these minimizers remain rigid for σ in a neighborhood of n. We conclude with numerical evidence of a phase transition phenomenon, where the location of the minimum undergoes discrete jumps as the parameter σ varies.

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