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Contact projective structures and 5-dim para-CR manifolds with Levi form degenerate in 1-direction

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(1) 3rd order ODEs modulo contact transformations.

In paper with Godliński arXiv 0902.4129 we produced EDSs describing geometry of 3rd order ODE's, and in particular in the contact equivalence case we produced the following system:

$$
\text{(GN)}\quad
\begin{aligned}
d\theta^1 &= \Omega_1\theta^1 + \theta^4\theta^2\\
d\theta^2 &= \Omega_2\theta^1 + \Omega_3\theta^2 + \theta^4\theta^3\\
\longrightarrow\ d\theta^3 &= \Omega_2\theta^2 + (2\Omega_3-\Omega_1)\theta^3 + A_2\theta^2\theta^1 + \boxed{\boxed{A_1}\,\theta^4\theta^1}\\
d\theta^4 &= \Omega_4\theta^1 + \Omega_5\theta^2 + (\Omega_1-\Omega_3)\theta^4\\
d\Omega_1 &= \Omega_6\theta^1 + \Omega_4\theta^2 - \Omega_2\theta^4\\
d\Omega_2 &= (\Omega_3-\Omega_1)\Omega_2 + \tfrac12\Omega_6\theta^2 + \Omega_4\theta^3 + A_3\theta^1\theta^2 + A_4\theta^1\theta^4\\
d\Omega_3 &= \tfrac12\Omega_6\theta^1 + \Omega_4\theta^2 + \Omega_5\theta^3 + A_5\theta^1\theta^2 + A_2\theta^1\theta^4\\
d\Omega_4 &= \Omega_5\Omega_2 + \Omega_4\Omega_3 + \tfrac12\Omega_6\theta^4 + (A_6+B_2)\theta^1\theta^2 + 2B_3\theta^1\theta^3\\
&\quad - A_3\theta^1\theta^4 + B_4\theta^2\theta^3\\
\longrightarrow\ d\Omega_5 &= (\Omega_1-2\Omega_3)\Omega_5 + \Omega_4\theta^4 + (A_7+B_3)\theta^1\theta^2 + B_4\theta^1\theta^3 - A_5\theta^1\theta^4\\
&\quad \boxed{+\boxed{B_1}\,\theta^2\theta^3}\\
d\Omega_6 &= \Omega_6\Omega_1 + 2\Omega_4\Omega_2 + C_1\theta^1\theta^2 + 2B_2\theta^1\theta^3 + A_8\theta^1\theta^4 + 2B_3\theta^2\theta^3
\end{aligned}
$$

These are curvature conditions $d\omega + \omega\wedge\omega = K_{ij}\theta^i\wedge\theta^j$ for the Cartan connection

$$
\omega = \begin{pmatrix}
\tfrac12\Omega_1 & \tfrac12\Omega_2 & -\tfrac12\Omega_4 & -\tfrac14\Omega_6\\
\theta^4 & \Omega_3-\tfrac12\Omega_1 & -\Omega_5 & -\tfrac12\Omega_4\\
\theta^2 & \theta^3 & \tfrac12\Omega_1-\Omega_3 & -\tfrac12\Omega_2\\
2\theta^1 & \theta^2 & -\theta^4 & -\tfrac12\Omega_1
\end{pmatrix} \in \underline{sp}(4,\mathbb{R}) \simeq \underline{o}(2,3)
$$

The basic invariants ~~for~~ $y''' = F(x,y,y',y'')$, $y'=p$, $y''=q$ are:

Wunschmann

$\boxed{A_1} = (\ )\cdot[9D^2F_q - 27DF_p - 18DF_qF_q + 18F_pF_q + 4F_q^3 + 54F_y]$

$\boxed{B_1} = (\ )\ [F_{qqqq}]$ chern

All other A_i's are coframe derivatives of A_1

B_i's are coframe derivatives of B_1

and C_1 is a coframe derivative of A_1 and B_1