INTRODUCTION TO MOTIVIC COHOMOLOGY – A SUMMARY OF SERIES OF IMPANGA-LECTURES BY MAREK SZYJEWSKI

The theory of Grothendieck motives yields a universal functor from the category Sm_k of smooth varieties of finite type over a field k to a (pre)abelian category.

The Voevodsky motive category $\mathbf{DM}^{eff,-}_{Nis}(k,R)$ is the target for the motive functor $M : X \mapsto M(X)$. This is a monoidal functor from a symmetric monoidal category Sm_k to a tensor triangulated category.

Firstly, we construct a category Cor_k of finite correspondences which has the same objects as Sm_k and has "more" morphisms. The category Cor_k is a symmetric monoidal category with monoidal operation $\times$.

Next, we consider a category $\mathbf{PST}_k$ of "presheaves with transfers" on Cor_k, i.e. the category of contravariant functors from Cor_k to abelian groups. $\mathbf{PST}_k$ is an abelian category with enough projectives and injectives. Moreover, an object X of Cor_k or Sm_k corresponds to a projective object $\mathbb{Z}_{tr}(X)$ of $\mathbf{PST}_k$.

There is several tasks to do with $\mathbf{PST}_k$:

- to distinguish the subcategory of homotopy invariant presheaves;
- to define motivic complexes $\mathbb{Z}(q) = C_*\left(\mathbb{Z}_{tr}(\mathbb{G}_m^{\wedge q})\right)[-q]$ and a pairing $\mathbb{Z}(p)\otimes \mathbb{Z}(q) \to \mathbb{Z}(p+q)$;
- to compute as much of their cohomology as possible; e.g. $H^p(SpecF, \mathbb{Z}(p)) = K^M_p(F)$ sets affirmative the Beilinson conjecture;
- to study the sheafication functors with respect to Zariski, Nisnevich and étale topology on homotopy invariant presheaves (e.g. the Zariski sheafication yields the same sheaves as the Nisnevich sheafication);
- to produce a monoidal functor to a tensor triangulated category $\mathbf{DM}^{eff,-}_{Nis}(k,R)$, such that $M(X\times Y) = M(X)\otimes M(Y)$;
- to provide étale analogues of the above.

Lectures are based on "Notes on Motivic Cohomology" by C. Mazza, V. Voevodski and C. Weibel.

The first lecture: 25.04.2003, IMPAN, room 403, 10:30–12:00.