

# Metrically universal Polish group

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We show that for any positive real  $K$  there is a Polish group  $\mathbb{G}_K$  equipped with a bi-invariant metric bounded by  $K$  that is metrically universal for all separable groups equipped with bi-invariant metric bounded by  $K$ . In other words, every separable group equipped with a bi-invariant metric bounded by  $K$  is isometrically isomorphic to a subgroup of  $\mathbb{G}_K$ .

In particular, there exists a universal object in the class of Polish groups admitting a compatible bi-invariant metric.

On the other hand, we show that there is no metrically universal separable group with bi-invariant unbounded metric. In other words, for every separable group  $G$  equipped with a bi-invariant metric there is another separable group  $H$  equipped with a bi-invariant metric such that  $H$  is not isometrically isomorphic to any subgroup of  $G$ .