

Haar null sets without Haar measure

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It is well-known that if a Polish group is not locally compact (think of e.g. $C[0, 1]$ or S_∞) then there is no nice invariant measure on it. However, as Christensen showed, the notion of a *nullset* has a very natural generalisation, hence it makes sense to talk about almost every continuous function or almost every permutation of ω . In this talk we will answer an old problem of Mycielski by showing that not every nullset has a G_δ hull. We also answer a question from Fremlin's list by showing that there exists a universally measurable (actually, co-analytic) nullset without a Borel hull. We will also touch on various other problems concerning the behaviour of almost every continuous function in the above sense, and also state numerous set-theoretical open problems concerning this σ -ideal.