

THE HUREWICZ DICHOTOMY FOR GENERALIZED BAIRE SPACES

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ABSTRACT. An analytic subset of a Polish space X is not covered by a K_σ subset of X if and only if it contains a closed subset homeomorphic to the Baire space ${}^\omega\omega$ by a result of Hurewicz. We consider the analogous statement (the *Hurewicz dichotomy*) for Σ_1^1 subsets of the generalized Baire space ${}^\kappa\kappa$, for uncountable regular cardinals κ with $\kappa^{<\kappa} = \kappa$. One of the main results shows that if GCH holds, then there is a cardinal preserving extension in which the Hurewicz dichotomy for Σ_1^1 subsets of ${}^\kappa\kappa$ holds at all uncountable regular cardinals, while strongly unfoldable and supercompact cardinals are preserved. On the other hand in L , and after adding a Cohen real to a model of GCH, the Hurewicz dichotomy for Σ_1^1 subsets of ${}^\kappa\kappa$ fails at all uncountable regular cardinals. We also consider connections with the perfect set property and κ -Miller measurability. This is joint work with Philipp Lücke and Luca Motto Ros.

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