

ADDITIVE GROUP ACTIONS ON ALGEBRAIC VARIETIES

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Let X be an algebraic variety over the base field k . The additive group \mathbb{G}_a is the base field $(k, +)$ seen as algebraic group with its additive structure. In this mini-course we will give an overview of the classification of actions of the additive group by means of certain integrable vector fields on X .

The following plan is subject to changes depending on the interests and background of the audience. If for some reason you did not attend the first lecture, you can still enter the mini-course. The first lecture is not prerequisite for the last two lectures.

- In the first lecture we will develop the theory of \mathbb{G}_a -actions on affine varieties. In this case, \mathbb{G}_a -actions are in one to one correspondence with vector fields satisfying a nilpotency condition. This subject is classical from affine geometry, see for instance [Fre06].
- In the second lecture we will generalize the classification of \mathbb{G}_a -actions by means of vector fields to a wide class of varieties including projective varieties. This is a joint work with Adrien Dubouloz [DL16].
- Finally, in the third lecture we will show how these techniques, in the presence of a torus action, allow for the systematic study of root subgroups. A root subgroup is a closed subgroups of $\text{Aut}(X)$ isomorphic to \mathbb{G}_a that is normalized by the torus. In particular, we will recover the description given in [Dem70] of the connected component of the automorphism group of a complete toric variety, see also [AHHL14].

REFERENCES

- [AHHL14] Ivan Arzhantsev, Jürgen Hausen, Elaine Herppich, and Alvaro Liendo. The automorphism group of a variety with torus action of complexity one. *Mosc. Math. J.*, 14(3):429–471, 641, 2014.
- [Dem70] Michel Demazure. Sous-groupes algébriques de rang maximum du groupe de Cremona. *Ann. Sci. École Norm. Sup. (4)*, 3:507–588, 1970.
- [DL16] Adrien Dubouloz and Alvaro Liendo. Rationally integrable vector fields and rational additive group actions. *Internat. J. Math.*, 27 (2016), no. 8, 1650060, 19 pp.
- [Fre06] Gene Freudenburg, *Algebraic theory of locally nilpotent derivations.*, Encyclopaedia of Mathematical Sciences 136. Invariant Theory and Algebraic Transformation Groups 7. Berlin: Springer, 2006.

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