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The convolution of measures
in the model of particles collisions

Our goal is to present new sufficient conditions for the asymptotic stability of Markov operators with respect to the total variation norm, acting on the space of signed measures. Our results are based on two principles. The first one is the LaSalle invariance principle used in the theory of dynamical systems (see [C]). The second is related to some special property of the total variation norm which we call the maximum principle. This approach is strongly related to the classical results concerning differential equations on convex subsets of Banach spaces (see [C]).

To illustrate the application of our results we will discuss the problem of the asymptotic stability of solutions of a generalized version of the Tjon–Wu equation (see [TW]), related to the kinetic theory of gases. The maximum principle method in studying the asymptotic stability of Markov semigroup with respect to other metrics, for example the Kantorovich–Wasserstein metric and Fortet–Mourier metric was used in the papers [LT1], [G1] and [G2].

The convex sets method in studying the Boltzmann equation was used in a series of papers (see for example: [G2, L1, L2, LT1, LT2]).

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