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The Goursat problem and hodograph method

The problem of mass transport by an electric field (zone electrophoresis) is studied. Mathematical model is the two conservation laws with discontinuous initial data. The discontinuities are located in different space points. The interactions of the weak and strong discontinuities are investigated. Three type of interactions are possible: shock-shock waves, shock-rarefaction waves, and rarefaction-rarefaction waves. The first and the second type of interactions is described in [1]. In the case of the rarefaction-rarefaction waves interaction we have the Goursat problem with initial date on characteristics. To solve this problem the variant of the hodograph method is used (see [2]–[4]). The linear system with variable coefficients is constructed. Using the solvability conditions we transform the original conservation laws to linear hyperbolic PDE of the second order with variable coefficients. For this equation the Riemann-–Green function is well-known. The implicit analytical solution of the Goursat problem is constructed. To obtain the explicit form of the solution we use the special analytical and numerical methods which allow us to solve the zonal electrophoresis equations with discontinuous initial data.

References

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