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On the numerical solution of an ODE using an orthogonal expansion

We treat here a differential equation of an arbitrary order m

$$\sum_{r=0}^{m} a_{m-r}(x) y^{(m-r)}(x) = f(x) \tag{1}$$

where we expand both the known function $f(x)$ and the unknown function $y(x)$ by orthogonal polynomials

$$f(x) = \sum_{j=0}^{n} f_j \phi_j(x) m \leqslant n \tag{2}$$

$$f_j = \frac{(f, \phi_j)_{L^2}}{||\phi_j||^2_{L^2}} \qquad \forall\, 0 \leqslant j \leqslant n \tag{3}$$

$$y(x) = \sum_{j=0}^{n} y_j \phi_j(x) \tag{4}$$

$$y_j = \frac{(y, \phi_j)_{L^2}}{||\phi_j||^2_{L^2}} \qquad \forall\, 0 \leqslant j \leqslant n \tag{5}$$

After setting these expansions to the differential equation (1) and using the Galerkin Method we reduce the problem to a linear algebraic problem easy to treat it numerically. Such expansions leads us to an equations system of order at least $m+1$, where the RHS is very easy to obtain. In general case it is not necessary to impose any conditions to the solution of (1).