On the numerical solution of an ODE using an orthogonal expansion

We treat here a differential equation of an arbitrary order \( m \)

\[
\sum_{r=0}^{m} a_{m-r}(x)y^{(m-r)}(x) = f(x)
\]

(1)

where we expand both the known function \( f(x) \) and the unknown function \( y(x) \) by orthogonal polynomials

\[
f(x) = \sum_{j=0}^{n} f_j \phi_j(x) m \leq n
\]

(2)

\[
f_j = \frac{(f, \phi_j)_{L^2}}{\|\phi_j\|_{L^2}^2} \quad \forall \ 0 \leq j \leq n
\]

(3)

\[
y(x) = \sum_{j=0}^{n} y_j \phi_j(x)
\]

(4)

\[
y_j = \frac{(y, \phi_j)_{L^2}}{\|\phi_j\|_{L^2}^2} \quad \forall \ 0 \leq j \leq n
\]

(5)

After setting these expansions to the differential equation (1) and using the Galerkin Method we reduce the problem to a linear algebraic problem easy to treat numerically. Such expansions leads us to an equations system of order at least \( m+1 \), where the RHS is very easy to obtain. In general case it is not necessary to impose any conditions to the solution of (1).