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Stability of fixed points in an approximate solution of the spring-mass running model

We consider a classical spring-mass model of human running which is built upon an inverted elastic pendulum (see the seminal papers [2,4]). Based on our previous results concerning asymptotic solutions for large spring constant (or small angle of attack), we construct analytical approximations of solutions in the considered model based on the perturbation theory (see [5] and compare with [3]).

The model itself consists of two sets of differential equations — one set describes the motion of the centre of mass of a runner in contact with the ground (support phase), and the second set describes the phase of no contact with the ground (flight phase).

By appropriately concatenating asymptotic solutions for the two phases we are able to reduce the dynamics to a one-dimensional apex to apex return map. We find sufficient conditions for this map to have a unique stable fixed point. By numerical continuation of fixed points with respect to energy, we find a transcritical bifurcation in our model system.

Bibliografia

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