On the Born-Infeld equation: external charges and nonlinearities

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It is well known that, in the classical Maxwell electromagnetic theory, the so called *infinite-energy problem* appears: the energy of the electrostatic field generated by a pointwise charge is infinite. In the first years of the past century, Born and Infeld proposed to solve such a problem introducing a nonlinear modification of the Maxwell theory that, in the electrostatic case, gives rise to an equation which replaces the Gauss law (or Poisson equation) and that involves the mean curvature operator in the Lorentz-Minkowski metric.

The aim of this talk, therefore, is to introduce the Born-Infeld equation, showing its main peculiarities.

In the first part, we deal, briefly, with

$$\begin{cases} -\operatorname{div}\left(\dfrac{\nabla\phi}{\sqrt{1-|\nabla\phi|^2}}\right) = \rho, \quad x \in \mathbb{R}^N, \\ \lim\limits_{|x|\to\infty} \phi(x) = 0, \end{cases} \tag{1}$$

where ϕ is the electric potential and ρ is an assigned extended charge density. We present some existence, uniqueness and regularity of the solution of (1).

In the second part, instead, we deal with

$$\begin{cases} -\operatorname{div}\left(\dfrac{\nabla\phi}{\sqrt{1-|\nabla\phi|^2}}\right) = g(\phi), \quad x \in \mathbb{R}^N, \\ \lim\limits_{|x|\to\infty} \phi(x) = 0, \end{cases} \tag{2}$$

where g is a continuous nonlinearity. We present some recent existence results for problem (2) for a large class of nonlinearities by means of variational methods or ODE techniques.