Abstract

In this thesis, we investigate zigzags in triangulations of surfaces. We introduce the concept of z-monodromy and show that there are precisely 7 types (M1)–(M7) of z-monodromies of faces in triangulations. We provide examples for all these types.

A triangulation is z-knotted (i.e. it contains a single zigzag up to reversing) if and only if the z-monodromy of each face is of one of the type (M1)–(M4). Using this fact we show that each triangulation admits a z-knotted shredding. The proof is constructive.

Another result related to z-monodromies which we prove states that the z-monodromies (M1) and (M2) are exceptional. For each $i \geq 3$ there is a triangulation with the z-monodromy of type (Mi) for all faces. For (M1) and (M2) this fails: all faces with the z-monodromy of one of these types form a forest in the dual graph.

We investigate z-oriented triangulations, i.e. triangulations with a direction chosen on each zigzag. There are precisely two types of faces in such triangulations. We show that each z-oriented triangulation admits a z-oriented shredding with all faces of the first type. We will focus only on such triangulations. An important subclass is formed by so called z-homogeneous triangulations. We describe a one-to-one correspondence between z-homogeneous triangulations and embeddings of Eulerian digraphs in surfaces. We show that a z-oriented triangulation (with all faces of the first type) provide a decomposition of the surface into connected components of the following three types: open discs, open cylinders and open Möbius strips. The triangulation is z-homogeneous if and only if all connected components are open discs.

Since z-knotted triangulations have a single z-orientation (up to reversing), we can say on z-homogeneity of z-knotted triangulations without fixing a z-orientation. We propose an algorithm of constructing of such a triangulation from an arbitrary z-homogeneous triangulation. This construction is based on the z-monodromies of pairs of edges.