INSTITUTE OF MATHEMATICS of the Polish Academy of Sciences

ul. Śniadeckich 8, P.O.B. 21, 00-956 Warszawa 10, Poland http://www.impan.pl

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Joanna Janczewska

Homoclinic solutions for a class of autonomous second order Hamiltonian systems with a superquadratic potential

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Abstract

We will prove the existence of a nontrivial homoclinic solution for an autonomous second order Hamiltonian system $\ddot{q} + \nabla V(q) = 0$, where $q \in \mathbb{R}^n$, a potential $V \colon \mathbb{R}^n \to \mathbb{R}$ is of the form $V(q) = -K(q) + W(q)$, K and W are C^1-maps, K satisfies the pinching condition, W grows at a superquadratic rate, as $|q| \to \infty$ and $W(q) = o(|q|^2)$, as $|q| \to 0$. A homoclinic solution will be obtained as a weak limit in the Sobolev space $W^{1,2}(\mathbb{R}, \mathbb{R}^n)$ of a sequence of almost critical points. For this purpose, we will apply a general minimax principle to the corresponding action functional.

key words and phrases: action functional, Hamiltonian system, homoclinic solution, general minimax principle, superquadratic potential

AMS Subject Classification: 37J45 (58E05, 34C37, 70H05)

1 Introduction

This paper concerns the existence of homoclinic solutions for a certain class of autonomous second order Hamiltonian systems. Let us consider

$$\ddot{q} + \nabla V(q) = 0, \tag{1}$$

where $q \in \mathbb{R}^n$ and a potential $V \colon \mathbb{R}^n \to \mathbb{R}$ satisfies the following conditions:

(H_1) $V(q) = -K(q) + W(q)$, where $K, W \colon \mathbb{R}^n \to \mathbb{R}$ are C^1-maps,

(H_2) there are constants $b_1, b_2 > 0$ such that for all $q \in \mathbb{R}^n$,

$$b_1|q|^2 \leq K(q) \leq b_2|q|^2,$$

(H_3) $(q, \nabla K(q)) \leq 2K(q)$ for all $q \in \mathbb{R}^n$,

(H_4) $2K(q) - (q, \nabla K(q)) = o(|q|^2)$, as $|q| \to 0$,

(H_5) ∇K is Lipschitzian in a neighbourhood of $0 \in \mathbb{R}^n$,

(H_6) $\nabla W(q) = o(|q|)$, as $|q| \to 0$,

(H_7) there is a constant $\mu > 2$ such that for every $q \in \mathbb{R}^n \setminus \{0\}$,

$$0 < \mu W(q) \leq (q, \nabla W(q)).$$

Here and subsequently, $(\cdot,\cdot)\colon \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ denotes the standard inner product in $\mathbb{R}^n$ and $|\cdot|\colon \mathbb{R}^n \to [0,\infty)$ is the induced norm.

It is worth pointing out that if $K\colon \mathbb{R}^n \to \mathbb{R}$ is a C^2-map satisfying (H_2), then (H_4) takes place. Let us also remark that (H_6) and (H_7) imply

$$W(q) = o(|q|^2), \text{ as } |q| \to 0. \tag{2}$$

Moreover, from (H_7) it follows that for $q \neq 0$ a map given by

$$(0,\infty) \ni \zeta \longmapsto W(\zeta^{-1}q)\zeta^{\mu}$$

is nonincreasing. Hence the following inequalities hold

$$W(q) \leq W\left(\frac{q}{|q|}\right)|q|^{\mu} \text{ if } 0 < |q| \leq 1, \tag{3}$$

$$W(q) \geq W\left(\frac{q}{|q|}\right)|q|^{\mu} \text{ if } |q| \geq 1. \tag{4}$$

By (H_2) and (4) we get that a potential V grows at a superquadratic rate, as $|q| \to \infty$, i.e.

$$\frac{V(q)}{|q|^2} \to \infty, \quad \text{as } |q| \to \infty.$$

Hamiltonian systems with superquadratic potentials were also considered by V. Coti Zelati, I. Ekeland and E. Séré in [4], H. Hofer and K. Wysocki in [7], V. Coti Zelati and P. Rabinowitz in [5], P. Rabinowitz and K. Tanaka in [14], W. Omana and M. Willem in [11], Xiangjin Xu in [16].

It is easily seen that $q \equiv 0$ is a solution of (1). In this work we are interested in the existence of nontrivial homoclinic solutions of (1) that emanate from 0 and terminate at 0, i.e.

$$\lim_{t\to\pm\infty} q(t) = q(\pm\infty) = 0.$$

The existence of homoclinic orbits for first and second order Hamiltonian systems has been studied by many authors and the literature on this subject is vast (see [1,2,6,8,9,12,15]), but many questions are still open (see the survey [13] by P. Rabinowitz). Finding homoclinic solutions in Hamiltonian systems can be quite difficult. In the last 20 years, a great progress was made by applying variational methods (see the survey [3] by T. Bartsch and A. Szulkin). For instance, the authors of [4] studied a class of first order Hamiltonian systems using a dual variational transformation and the Mountain Pass Theorem to prove the existence of two distinct homoclinic solutions. P. Rabinowitz in [12] examined a family of second order Hamiltonian systems applying the Mountain Pass Theorem to get a sequence of subharmonic solutions and suitable estimates to pass to a nontrivial limit which occurred to be a nontrivial homoclinic solution (see also [2,8,9]).

The theorem which we shall prove is as follows.

Theorem 1.1 *If the assumptions $(H_1)-(H_7)$ are satisfied then the Hamiltonian system* (1) *possesses a nontrivial homoclinic solution $q_0 \in W^{1,2}(\mathbb{R},\mathbb{R}^n)$ such that $\dot{q}_0(\pm\infty) = 0$.*

□

This result is proved in Section 2 by studying the corresponding to (1) action functional $I\colon W^{1,2}(\mathbb{R},\mathbb{R}^n) \to \mathbb{R}$. Applying a general minimax principle (see Theorem 2.3) we receive a sequence $\{q_k\}_{k\in\mathbb{N}}$ such that $\{I(q_k)\}_{k\in\mathbb{N}}$ is bounded and $I'(q_k) \to 0$, as $k \to \infty$. We show that $\{q_k\}_{k\in\mathbb{N}}$ has a weakly convergent subsequence and its weak limit is a desired homoclinic solution.

A general minimax principle which is a consequence of Ekeland's variational principle (see Theorem 4.1 in [10]) was also applied by P. Rabinowitz and K. Tanaka in Section 5 of [14]. We are partially motivated by [14]. Their problem is completely different from ours, but the proofs of the existence of almost critical points are similar. However, there are some new tricks involved in this manuscript. For example, to get a nontrivial homoclinic orbit before passing to a weak limit with a sequence of almost critical points each element of this sequence has to be appropriately shifted.

2 Proof of Theorem 1.1

The proof of Theorem 1.1 will be divided into a sequence of lemmas. Let E be the Sobolev space $W^{1,2}(\mathbb{R},\mathbb{R}^n)$ with the standard norm

$$\|q\|_E := \left(\int_{-\infty}^{\infty} \left(|q(t)|^2 + |\dot{q}(t)|^2\right) dt\right)^{\frac{1}{2}}.$$

We first recall two elementary inequalities concerning functions in E.

Fact 2.1 *If $q\colon \mathbb{R} \to \mathbb{R}^n$ is a continuous mapping such that $\dot{q} \in L^2_{loc}(\mathbb{R},\mathbb{R}^n)$, then for every $t \in \mathbb{R}$,*

$$|q(t)| \leq \sqrt{2}\left(\int_{t-\frac{1}{2}}^{t+\frac{1}{2}} \left(|q(s)|^2 + |\dot{q}(s)|^2\right) ds\right)^{\frac{1}{2}}. \tag{5}$$

□

The proof of Fact 2.1 can be found in [8]. (See Fact 2.8, p. 385.)

Fact 2.2 *For each $q \in E$,*

$$\|q\|_{L^\infty(\mathbb{R},\mathbb{R}^n)} \leq \sqrt{2}\|q\|_E. \tag{6}$$

□

Fact 2.2 is a direct consequence of the inequality (5).

Let $I\colon E \to \mathbb{R}$ be given by

$$I(q) := \int_{-\infty}^{\infty} \left[\frac{1}{2}|\dot{q}(t)|^2 - V(q(t))\right] dt.$$

By $(H_5) - (H_7)$ it is obvious that $I \in C^1(E,\mathbb{R})$. Moreover,

$$I'(q)w = \int_{-\infty}^{\infty} \left[(\dot{q}(t),\dot{w}(t)) - (\nabla V(q(t)),w(t))\right] dt$$

for all $q, w \in E$ and any critical point of I on E is a classical solution of (1) with $q(\pm\infty) = 0$, as is easy to verify. In order to prove Theorem 1.1, we apply a general minimax principle. Let us remind it.

Theorem 2.3 (see Theorem 4.3 in [10]) *Let K be a compact metric space, $K_0 \subset K$ a closed subset, X a Banach space and $\chi \in C(K_0, X)$. Let $\mathcal{M}$ be a complete metric space given by*

$$\mathcal{M} := \{g \in C(K, X) \colon g(s) = \chi(s) \quad \textit{if} \quad s \in K_0\}$$

with the usual distance. Let $\varphi \in C^1(X, \mathbb{R})$ and let us define

$$c = \inf_{g \in \mathcal{M}} \max_{s \in K} \varphi(g(s)),$$

$$c_1 = \max_{\chi(K_0)} \varphi.$$

If $c > c_1$ then for each $\varepsilon > 0$ and for each $h \in \mathcal{M}$ such that

$$\max_{s \in K} \varphi(h(s)) \leq c + \varepsilon$$

there exists $v \in X$ such that

$$c - \varepsilon \leq \varphi(v) \leq \max_{s \in K} \varphi(h(s)),$$

$$dist(v, h(K)) \leq \varepsilon^{\frac{1}{2}},$$

$$\|\varphi'(v)\|_{X^*} \leq \varepsilon^{\frac{1}{2}}.$$

$\square$

Set

$$\bar{b}_1 := \min\{1, 2b_1\},$$

$$\bar{b}_2 := \max\{1, 2b_2\},$$

where b_1, b_2 are the constants of the pinching condition (H_2). By definition, $\bar{b}_1 \leq 1 \leq \bar{b}_2$. From (H_2) we have

$$I(q) \geq \frac{1}{2}\bar{b}_1 \|q\|_E^2 - \int_{-\infty}^{\infty} W(q(t))dt \tag{7}$$

for every $q \in E$. By (2), (6) and (7), we conclude that there are constants $\alpha, \varrho > 0$ such that

$$I(q) \geq \alpha, \quad \text{if} \quad \|q\|_E = \varrho. \tag{8}$$

Take $\nu \in C_0^\infty(\mathbb{R}, \mathbb{R}^n)$ such that $|\nu(t)| = 1$ for $|t| \leq 1$ and $\nu(t) = 0$ for $|t| \geq 2$. Set

$$m := \inf\{W(q) \colon |q| = 1\}.$$

From (4), for every $\xi \in \mathbb{R}$ such that $|\xi| \geq 1$, we have

$$\begin{aligned} \int_{-\infty}^{\infty} W(\xi\nu(t))dt &\geq \int_{-1}^{1} W(\xi\nu(t))dt \\ &\geq \int_{-1}^{1} W\left(\frac{\xi\nu(t)}{|\xi\nu(t)|}\right) |\xi\nu(t)|^\mu dt \geq 2m|\xi|^\mu. \end{aligned}$$

Combining this with (H_2) we obtain

$$I(\xi\nu) \leq \frac{1}{2}\bar{b}_2\xi^2\|\nu\|_E^2 - 2m|\xi|^\mu.$$

Since $m > 0$ and $\mu > 2$, for $|\xi|$ sufficiently large, $I(\xi\nu) < 0$. Consequently, there exists $Q \in E$ such that

$$\|Q\|_E > \varrho \quad \text{and} \quad I(Q) < 0 = I(0). \tag{9}$$

From now on, let

$$\mathcal{M} := \{g \in C([0,1], E)\colon g(0) = 0 \;\text{ and }\; g(1) = Q\} \tag{10}$$

and

$$c := \inf_{g\in\mathcal{M}} \max_{s\in[0,1]} I(g(s)). \tag{11}$$

By $(8) - (11)$, we get

$$c \geq \alpha > 0.$$

Applying Theorem 2.3 we conclude that the following lemma holds.

Lemma 2.4 *There exists a sequence $\{q_k\}_{k\in\mathbb{N}}$ in E such that*

$$I(q_k) \to c \quad \text{and} \quad I'(q_k) \to 0, \tag{12}$$

as $k \to \infty$.

□

Lemma 2.5 *The sequence $\{q_k\}_{k\in\mathbb{N}}$ given by* (12) *is bounded in E.*

Proof. By (12), for large k,

$$\|I'(q_k)\|_{E^*} < 2 \quad \text{and} \quad |I(q_k) - c| < 1. \tag{13}$$

Applying (H_3) and (H_7) we obtain

$$I(q_k) - \frac{1}{2}I'(q_k)q_k \geq \left(\frac{\mu}{2} - 1\right)\int_{-\infty}^{\infty} W(q_k(t))dt \tag{14}$$

for $k \in \mathbb{N}$. Combining (14) with (13) we receive

$$c + 1 + \|q_k\|_E \geq \left(\frac{\mu}{2} - 1\right)\int_{-\infty}^{\infty} W(q_k(t))dt$$

for large k, and hence

$$\int_{-\infty}^{\infty} W(q_k(t))dt \leq \frac{2}{\mu - 2}\left(c + 1 + \|q_k\|_E\right). \tag{15}$$

By the use of (H_2), (H_3) and (H_7), we get

$$I'(q_k)q_k \leq \bar{b}_2\|q_k\|_E^2 - \mu\int_{-\infty}^{\infty} W(q_k(t))dt \tag{16}$$

for $k \in \mathbb{N}$. From (7) and (16) it follows that

$$\text{(17)}\quad \frac{1}{\bar{b}_1} I(q_k) - \frac{1}{\mu \bar{b}_2} I'(q_k) q_k \geq \left(\frac{1}{2} - \frac{1}{\mu}\right) \|q_k\|_E^2 - \left(\frac{1}{\bar{b}_1} - \frac{1}{\bar{b}_2}\right) \int_{-\infty}^{\infty} W(q_k(t)) dt$$

for $k \in \mathbb{N}$. By (13) and (17), for large k,

$$\text{(18)}\quad \frac{1}{\bar{b}_1} (c+1) + \|q_k\|_E \geq \left(\frac{1}{2} - \frac{1}{\mu}\right) \|q_k\|_E^2 - \left(\frac{1}{\bar{b}_1} - \frac{1}{\bar{b}_2}\right) \int_{-\infty}^{\infty} W(q_k(t)) dt.$$

Finally, from (15) and (18), for large k,

$$\text{(19)}\quad \begin{aligned} \left(\frac{1}{2} - \frac{1}{\mu}\right) \|q_k\|_E^2 &\leq \frac{1}{\bar{b}_1} (c+1) + \|q_k\|_E \\ &+ \frac{2}{\mu - 2} \left(\frac{1}{\bar{b}_1} - \frac{1}{\bar{b}_2}\right) (c + 1 + \|q_k\|_E). \end{aligned}$$

Since $\mu > 2$, (19) shows that $\{q_k\}_{k\in\mathbb{N}}$ is bounded in E.

□

For each $k \in \mathbb{N}$ there is $\tau_k \in \mathbb{R}$ such that a map $q_{\tau_k} : \mathbb{R} \to \mathbb{R}^n$ given by

$$q_{\tau_k}(t) := q_k(t + \tau_k),$$

where $t \in \mathbb{R}$, achieves a maximum at $0 \in \mathbb{R}$, i.e.

$$\text{(20)}\quad \max\{|q_{\tau_k}(t)| : t \in \mathbb{R}\} = |q_{\tau_k}(0)|.$$

Then $q_{\tau_k} \in E$. Applying a change of variables $t = s - \tau_k$, $dt = ds$, we obtain $\|q_{\tau_k}\|_E = \|q_k\|_E$, $I(q_{\tau_k}) = I(q_k)$ and $\|I'(q_{\tau_k})\|_{E^*} = \|I'(q_k)\|_{E^*}$, as is easy to check. In consequence, by Lemma 2.4,

$$\text{(21)}\quad I(q_{\tau_k}) \to c \quad \text{and} \quad I'(q_{\tau_k}) \to 0,$$

as $k \to \infty$, and by Lemma 2.5, the sequence $\{q_{\tau_k}\}_{k\in\mathbb{N}}$ is bounded in E. Since E is a reflexive Banach space, $\{q_{\tau_k}\}_{k\in\mathbb{N}}$ possesses a weakly convergent subsequence in E.

Let q_0 denote a weak limit of a weakly convergent subsequence of $\{q_{\tau_k}\}_{k\in\mathbb{N}}$. Without loss of generality, we will write

$$\text{(22)}\quad q_{\tau_k} \rightharpoonup q_0 \ \text{ in } E,$$

as $k \to \infty$, which implies $q_{\tau_k} \to q_0$ in $L^\infty_{\text{loc}}(\mathbb{R}, \mathbb{R}^n)$, as $k \to \infty$.

Lemma 2.6 *q_0 given by (22) is a homoclinic solution of (1).*

Proof. Since $q_0 \in E$, we see that $q_0(t) \to 0$, as $t \to \pm\infty$, by Fact 2.1. Therefore, it is sufficient to show that $I'(q_0) = 0$. Fix $w \in C_0^\infty(\mathbb{R}, \mathbb{R}^n)$ and assume that for some $A > 0$, $\operatorname{supp}(w) \subset [-A, A]$. We have

$$I'(q_{\tau_k})w = \int_{-A}^{A} [(\dot{q}_{\tau_k}(t), \dot{w}(t)) - (\nabla V(q_{\tau_k}(t)), w(t))] \, dt$$

for each $k \in \mathbb{N}$. From (21) it follows that $I'(q_{\tau_k})w \to 0$, as $k \to \infty$. On the other hand,

$$\int_{-A}^{A} (\dot{q}_{\tau_k}(t), \dot{w}(t))dt \to \int_{-A}^{A} (\dot{q}_0(t), \dot{w}(t))dt,$$

as $k \to \infty$, by (22), and

$$\int_{-A}^{A} (\nabla V(q_{\tau_k}(t)), w(t))dt \to \int_{-A}^{A} (\nabla V(q_0(t)), w(t))dt,$$

as $k \to \infty$, because $q_{\tau_k} \to q_0$ uniformly on $[-A, A]$. Thus $I'(q_{\tau_k})w \to I'(q_0)w$, as $k \to \infty$, and, in consequence, $I'(q_0)w = 0$. Since $C_0^\infty(\mathbb{R}, \mathbb{R}^n)$ is dense in E, we get $I'(q_0) = 0$.

□

Lemma 2.7 *Let q_0 be given by* (22). *Then $\dot{q}_0(t) \to 0$, as $t \to \pm\infty$.*

Proof. From Fact 2.1, we obtain

$$|\dot{q}_0(t)|^2 \leq 2\int_{t-\frac{1}{2}}^{t+\frac{1}{2}} |\ddot{q}_0(s)|^2 ds + 2\int_{t-\frac{1}{2}}^{t+\frac{1}{2}} \left(|q_0(s)|^2 + |\dot{q}_0(s)|^2\right) ds.$$

For this reason, it suffices to notice that

$$\int_{r}^{r+1} |\ddot{q}_0(s)|^2 ds \to 0,$$

as $r \to \pm\infty$. Since q_0 satisfies (1), we have

$$\int_{r}^{r+1} |\ddot{q}_0(s)|^2 ds = \int_{r}^{r+1} |\nabla V(q_0(s))|^2 ds.$$

Take $\varepsilon > 0$. By (H_5) and (H_6), there is $\eta > 0$ such that for $|q| < \eta$, $|\nabla V(q)| < \varepsilon$. Moreover, there is $\delta > 0$ such that, if $|s| > \delta$, then $|q_0(s)| < \eta$. Hence, if $|r| > \delta + 1$, then

$$\int_{r}^{r+1} |\nabla V(q_0(s))|^2 ds < \varepsilon^2,$$

which completes the proof.

□

To finish the proof of Theorem 1.1, we have to show that $q_0 \neq 0$.

On the contrary, suppose that $q_0 \equiv 0$. Consequently, we have $q_{\tau_k}(0) \to 0$, as $k \to \infty$. From (20) it follows that $q_{\tau_k} \to 0$ uniformly on $\mathbb{R}$, as $k \to \infty$. By (21) and the boundedness of $\{q_{\tau_k}\}_{k\in\mathbb{N}}$ in E, we get

$$2I(q_{\tau_k}) - I'(q_{\tau_k})q_{\tau_k} \to 2c > 0, \tag{23}$$

as $k \to \infty$. On the other hand, by (H_4), (H_6) and (2),

$$\begin{aligned} 2I(q_{\tau_k}) - I'(q_{\tau_k})q_{\tau_k} &= \int_{-\infty}^{+\infty} [(\nabla V(q_{\tau_k}(t)), q_{\tau_k}(t)) - 2V(q_{\tau_k}(t))]\, dt \\ &= \int_{-\infty}^{+\infty} [2K(q_{\tau_k}(t)) - (\nabla K(q_{\tau_k}(t)), q_{\tau_k}(t))]\, dt \\ &+ \int_{-\infty}^{+\infty} [(\nabla W(q_{\tau_k}(t)), q_{\tau_k}(t)) - 2W(q_{\tau_k}(t))]\, dt \to 0, \end{aligned}$$

as $k \to \infty$. Indeed. Take $\varepsilon > 0$. From (H_4), (H_6) and (2), we deduce that there is $\delta > 0$ such that if $|q| < \delta$, then $|2K(q) - (\nabla K(q), q)| \leq \varepsilon |q|^2$, $|\nabla W(q)| \leq \varepsilon |q|$ and $|W(q)| \leq \varepsilon |q|^2$. Since $q_{\tau_k} \to 0$ uniformly on $\mathbb{R}$, there is $k_0 \in \mathbb{N}$ such that for $k > k_0$ and for $t \in \mathbb{R}$, $|q_{\tau_k}(t)| < \delta$. Hence $|2I(q_{\tau_k}) - I'(q_{\tau_k})q_{\tau_k}| \leq 4\varepsilon \|q_{\tau_k}\|_E^2$ for $k > k_0$, which contradicts (23).

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Institute of Mathematics of the Polish Academy of Sciences & Faculty of Technical Physics and Applied Mathematics, Gdańsk University of Technology, G. Narutowicza 11/12, 80-952 Gdańsk, Poland

E-mail addresses: j.janczewska@impan.pl, janczewska@mifgate.pg.gda.pl